On the Attenuation of Transient Fields by Imperfectly Conducting Spherical Shells

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Abstract—**Exact formulas for the electric and magnetic fields at any arbitrary point within a cavity region completely enclosed by a conducting spherical shell of arbitrary size are derived under the assumption that the exciting electromagnetic field is a linearly polarized, monochromatic, plane wave falling on the external surface of the shell. It is shown that the polarization of the electromagnetic field at the center of the cavity is the same as the polarization of the incident wave. From a knowledge of this steady-state solution, the time history of the electromagnetic field at the center of the cavity is calculated for the case where the incident wave is a Gaussian pulse. Numerical information on the effectiveness of the aluminum and copper shields under steady-state and transient conditions is provided for several pulse durations, shield sizes, and wall thicknesses.**

INTRODUCTION

A CLASS OF PROBLEMS known as shielding problems has recently been raised to a position of practical importance due to the requirement that certain field-sensitive devices be protected against the damaging effects of high-intensity electromagnetic pulses. Clearly, when an electromagnetic pulse falls on a shield, a relatively large part of the incident pulse is reflected by the outer surface of the shield and the remaining part is transmitted through the shield. The object of the shield is to keep the level of the transmitted part of the pulse below a prescribed level of safety.

The standard way of solving the problem is to take the temporal Fourier transform of the incident pulse and then to treat the problem as a steady-state boundary-value problem. The solution of the boundary-value problem yields the Fourier transform of the transmitted field in terms of the Fourier transform of the incident pulse. By taking the inverse Fourier transform of this solution, one obtains the time-dependent transmitted field as a function of the incident pulse. The one step of this procedure that limits its general applicability is the solving of the boundary-value problem. Unless the shield has a simple shape, the boundary-value problem cannot be solved with the completeness that the Fourier technique requires.

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In the present paper we accordingly choose the shell to be spherical because for a spherical shell the boundary-value problem can be solved exactly. We assume the incident wave to be a linearly polarized Gaussian pulse and calculate the resulting field at the center of the cavity. In the numerical work, pulses of several time durations and a number of shield dimensions are used. In particular, a sphere large enough to simulate a shielded room is considered.

PRELIMINARY REMARKS

Figure 1 illustrates a homogeneous imperfectly conducting spherical shell of outer radius a, inner radius b, and thickness $d(=b-a)$. The shell is characterized by permeability μ_1, dielectric constant ϵ_1, and conductivity σ_1. It is embedded in an infinite homogeneous medium with constitutive parameters μ_2, ϵ_2, and $\sigma_2=0$. The interior and exterior regions of the shield are assumed to possess the same electrical properties. The center of the shell is the origin of concentric Cartesian and spherical coordinate systems. The unit vectors in these systems are $\hat{x}$, $\hat{y}$, and $\hat{z}$; and $\hat{\theta}$, $\hat{\Phi}$, and $\hat{R}$, respectively. θ is the angle between $\hat{z}$ and $\hat{R}$, Φ is the angle between $\hat{x}$ and the projection of $\hat{R}$ in the xy plane, and R is measured from the origin. The incident electric field is linearly polarized

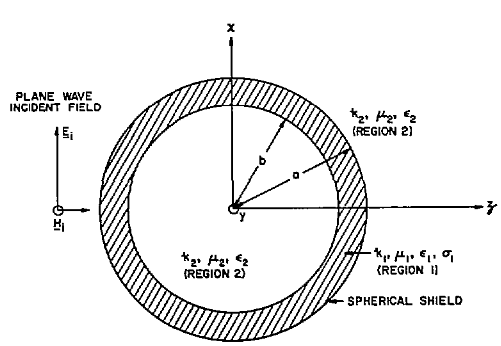

Fig. 1. Spherical shield. The fields E_i and H_i are polarized parallel to the x and y axes, respectively.

in the x direction and propagates in the direction of the positive z axis. It follows that the magnetic field is linearly polarized in the y direction. These fields are in time phase.

Mathematical Representation of the Electromagnetic Fields

The expansions, in vector spherical wave functions, of the incident, diffracted, shell, and cavity fields may be written down by analogy with the work of Stratton[1] on the problem of scattering from a solid imperfectly conducting sphere. The field expressions are

$$\boldsymbol{E}_i = \hat{x}E_0 e^{-jk_2 z} = E_0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left[\boldsymbol{m}_{\mathrm{o1n}}{}^{(1)} + j\boldsymbol{n}_{\mathrm{e1n}}{}^{(1)}\right] \qquad R \geq a \quad (1)$$

$$\boldsymbol{H}_i = -\frac{k_2}{\omega\mu_2} E_0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left[\boldsymbol{m}_{\mathrm{e1n}}{}^{(1)} - j\boldsymbol{n}_{\mathrm{o1n}}{}^{(1)}\right] \qquad R \geq a \quad (2)$$

$$\boldsymbol{E}_r = E_0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left[a_n{}^r \boldsymbol{m}_{\mathrm{o1n}}{}^{(3)*} + jb_n{}^r \boldsymbol{n}_{\mathrm{e1n}}{}^{(3)*}\right] \qquad R \geq a \quad (3)$$

$$\boldsymbol{H}_r = -\frac{k_2}{\omega\mu_2} E_0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \cdot \left[b_n{}^r \boldsymbol{m}_{\mathrm{e1n}}{}^{(3)*} - ja_n{}^r \boldsymbol{n}_{\mathrm{o1n}}{}^{(3)*}\right] \qquad R \geq a \quad (4)$$

$$\boldsymbol{E}_s = E_0 \left\{ \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left[p_n \boldsymbol{m}_{\mathrm{o1n}}{}^{(3)*} + jq_n \boldsymbol{n}_{\mathrm{e1n}}{}^{(3)*}\right] + \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left[d_n \boldsymbol{m}_{\mathrm{o1n}}{}^{(3)} + jf_n \boldsymbol{n}_{\mathrm{e1n}}{}^{(3)}\right] \right\} \qquad b \leq R \leq a \quad (5)$$

$$\boldsymbol{H}_s = -\frac{k_1}{\omega\mu_1} E_0 \left\{ \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \cdot \left[q_n \boldsymbol{m}_{\mathrm{e1n}}{}^{(3)*} - jp_n \boldsymbol{n}_{\mathrm{o1n}}{}^{(3)*}\right] + \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left[f_n \boldsymbol{m}_{\mathrm{e1n}}{}^{(3)} - jd_n \boldsymbol{n}_{\mathrm{o1n}}{}^{(3)}\right] \right\} \qquad b \leq R \leq a \quad (6)$$

$$\boldsymbol{E}_c = E_0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \left[a_n{}^c \boldsymbol{m}_{\mathrm{o1n}}{}^{(1)} + jb_n{}^c \boldsymbol{n}_{\mathrm{e1n}}{}^{(1)}\right] \qquad 0 \leq R \leq b \quad (7)$$

$$\boldsymbol{H}_c = -\frac{k_2}{\omega\mu_2} E_0 \sum_{n=1}^{\infty} (-j)^n \frac{2n+1}{n(n+1)} \cdot \left[b_n{}^c \boldsymbol{m}_{\mathrm{e1n}}{}^{(1)} - ja_n{}^c \boldsymbol{n}_{\mathrm{o1n}}{}^{(1)}\right]. \qquad 0 \leq R \leq b \quad (8)$$

The subscripts i, r, s, and c on the field vectors indicate incident, reflected, shell, and cavity, respectively. The time dependence assumed (and suppressed) in writing (1)–(8) is $\exp(j\omega t)$. The propagation constants in mediums 1 and 2 are

$$k_1 = \sqrt{\frac{\omega\mu_1\sigma_1}{2}}\,(1-j) \quad (9)$$

(for $\sigma_1 \gg \omega\epsilon_1$) and

$$k_2 = \omega\sqrt{\mu_2\epsilon_2}, \quad (10)$$

respectively. E_0 is the amplitude of the incident electric field. Also, $a_n{}^r$, $b_n{}^r$, p_n, q_n, d_n, f_n, $a_n{}^c$, and $b_n{}^c$ are constants to be evaluated from the boundary equations. The spherical vector wave functions are defined as follows:

$$\boldsymbol{m}_{\substack{\mathrm{e}\\\mathrm{o}}1n}^{(1)} = \pm \frac{j_n(kR)}{\sin\theta} P_n{}^1(\cos\theta) \frac{\cos\Phi}{\sin\Phi}\hat{\theta} - j_n(kR)\frac{\partial}{\partial\theta}P_n{}^1(\cos\theta)\frac{\sin\Phi}{\cos\Phi}\hat{\Phi} \quad (11)$$

$$\boldsymbol{n}_{\substack{\mathrm{e}\\\mathrm{o}}1n}^{(1)} = \frac{n(n+1)}{kR} j_n(kR) P_n{}^1(\cos\theta)\frac{\sin\Phi}{\cos\Phi}\hat{R} + \frac{1}{kR}[kRj_n(kR)]'\frac{\partial}{\partial\theta}P_n{}^1(\cos\theta)\frac{\sin\Phi}{\cos\Phi}\hat{\theta} \pm \frac{1}{kR\sin\theta}[kRj_n(kR)]'P_n{}^1(\cos\theta)\frac{\cos\Phi}{\sin\Phi}\hat{\Phi}. \quad (12)$$

$\boldsymbol{m}_{\substack{\mathrm{e}\\\mathrm{o}}1n}{}^{(3)}$ is obtained from $\boldsymbol{m}_{\substack{\mathrm{e}\\\mathrm{o}}1n}{}^{(1)}$ by writing $h_n{}^{(1)}(kR)$ for $j_n(kR)$ throughout the expression. $\boldsymbol{n}_{\substack{\mathrm{e}\\\mathrm{o}}1n}{}^{(3)}$ is obtained from $\boldsymbol{n}_{\substack{\mathrm{e}\\\mathrm{o}}1n}{}^{(1)}$ in like manner.

In writing down the expansions for the magnetic field, the relations

$$\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}, \quad (13)$$

and[2]

$$\left.\begin{aligned} \nabla \times \boldsymbol{m} &= k\boldsymbol{n} \\ \nabla \times \boldsymbol{n} &= k\boldsymbol{m} \\ \nabla \times \boldsymbol{m}^* &= k\boldsymbol{n}^* \\ \nabla \times \boldsymbol{n}^* &= k\boldsymbol{m}^* \end{aligned}\right\} \quad (14)$$

[1] J. A. Stratton, *Electromagnetic Theory*, 1st ed. New York: McGraw-Hill, 1941, p. 564.

[2] *Ibid.*, pp. 415–416.

are used. It is important to note that the notation $\boldsymbol{m}_{\substack{e\\o}1n}^{(3)*}$ and $\boldsymbol{n}_{\substack{e\\o}1n}^{(3)*}$ employed in this paper indicates that the complex conjugate of the function is to be taken. The argument of the function, even though complex, is to be left alone. Thus $h_n^{(1)*}(kR)\rightarrow h_n^{(2)}(kR)$ and $h_n^{(2)*}(kR)\rightarrow h_n^{(1)}(kR)$, where k may be complex.

The Electromagnetic Field at the Center of the Cavity

It can be shown that at the center of the spherical shell ($R=0$), (7) and (8) become

$$(\boldsymbol{E}_c)_{R=0} = E_0 b_1^c\{\sin\theta\cos\Phi\hat{R} + \cos\theta\cos\Phi\hat{\theta} - \sin\Phi\hat{\Phi}\} \tag{15}$$

$$(\boldsymbol{H}_c)_{R=0} = \frac{k_2}{\omega\mu_2} E_0 a_1^c\{\sin\theta\sin\Phi\hat{R} + \cos\theta\sin\Phi\hat{\theta} + \cos\Phi\hat{\Phi}\}. \tag{16}$$

Evidently an infinite sum of modes is not required to express the electric and magnetic fields at the middle of the sphere. Because of this fact the problem is greatly simplified. Since

$$\left.\begin{aligned}\hat{x} &= \sin\theta\cos\Phi\hat{R} + \cos\theta\cos\Phi\hat{\theta} - \sin\Phi\hat{\Phi}\\ \hat{y} &= \sin\theta\sin\Phi\hat{R} + \cos\theta\sin\Phi\hat{\theta} + \cos\Phi\hat{\Phi}\\ \hat{z} &= \cos\theta\hat{R} - \sin\theta\hat{\theta}\end{aligned}\right\} \tag{17}$$

it follows that

$$(\boldsymbol{E}_c)_{R=0} = \hat{x}E_0 b_1^c \tag{18}$$

$$(\boldsymbol{H}_c)_{R=0} = \hat{y}\frac{k_2}{\omega\mu_2} E_0 a_1^c. \tag{19}$$

Evidently the polarization of the cavity field is the same as the polarization of the incident field, and we have

$$\frac{|\boldsymbol{E}_c|}{|\boldsymbol{H}_c|_{R=0}} = \zeta\frac{|b_1^c|}{|a_1^c|} \tag{20}$$

where $\zeta=\omega\mu_2/k_2$.

The Boundary Equations

The boundary conditions that must be satisfied on the inner and outer surfaces of the spherical shield are

$$\left.\begin{aligned}(\boldsymbol{E}_s)_\theta &= (\boldsymbol{E}_c)_\theta\\ (\boldsymbol{E}_s)_\Phi &= (\boldsymbol{E}_c)_\Phi\\ (\boldsymbol{H}_s)_\theta &= (\boldsymbol{H}_c)_\theta\\ (\boldsymbol{H}_s)_\Phi &= (\boldsymbol{H}_c)_\Phi\end{aligned}\right\} R = b \tag{21}$$

$$\left.\begin{aligned}(\boldsymbol{E}_i + \boldsymbol{E}_r)_\theta &= (\boldsymbol{E}_s)_\theta\\ (\boldsymbol{E}_i + \boldsymbol{E}_r)_\Phi &= (\boldsymbol{E}_s)_\Phi\\ (\boldsymbol{H}_i + \boldsymbol{H}_r)_\theta &= (\boldsymbol{H}_s)_\theta\\ (\boldsymbol{H}_i + \boldsymbol{H}_r)_\Phi &= (\boldsymbol{H}_s)_\Phi\end{aligned}\right\} R = a. \tag{22}$$

It turns out that the θ and Φ boundary conditions give rise to two sets of redundant equations relating the constants. Hence, only the θ(or Φ) component of the fields at the boundaries are needed. In obtaining the simultaneous equations for the constants it is of importance to observe that $P_n^1(\cos\theta)$ and $(\partial/\partial\theta)P_n^1(\cos\theta)$ are different functions of θ and cannot be employed together in the same boundary equation.

Let the following notation be introduced:

$$\left.\begin{aligned}A_1 &= j_1(k_2a)\\ B_1 &= h_1^{(2)}(k_2a)\\ C_1 &= h_1^{(2)}(k_1a)\\ D_1 &= h_1^{(1)}(k_1a)\\ E_1 &= [k_2aj_1(k_2a)]'\\ F_1 &= [k_2ah_1^{(2)}(k_2a)]'\\ G_1 &= [k_1ah_1^{(2)}(k_1a)]'\\ H_1 &= [k_1ah_1^{(1)}(k_1a)]'\end{aligned}\right\}. \tag{23}$$

Also, let $A_2 \cdots H_2$ be defined in like manner except that b is written for a. It can then be shown that the boundary equations take the following form:

$$A_1 + a_1^rB_1 = p_1C_1 + d_1D_1 \tag{24}$$

$$\frac{E_1}{k_2} + \frac{b_1^r}{k_2}F_1 = \frac{q_1G_1}{k_1} + \frac{f_1H_1}{k_1} \tag{25}$$

$$\frac{k_2\mu_1}{k_1\mu_2}\{A_1 + b_1^rB_1\} = q_1C_1 + f_1D_1 \tag{26}$$

$$\frac{\mu_1}{\mu_2}\{E_1 + a_1^rF_1\} = p_1G_1 + d_1H_1 \tag{27}$$

$$p_1C_2 + d_1D_2 = a_1^cA_2 \tag{28}$$

$$\frac{q_1}{k_1}G_2 + \frac{f_1H_2}{k_1} = \frac{b_1^cE_2}{k_2} \tag{29}$$

$$\frac{\mu_2k_1}{\mu_1k_2}\{q_1C_2 + f_1D_2\} = b_1^cA_2 \tag{30}$$

$$\frac{\mu_2}{\mu_1}\{p_1G_2 + d_1H_2\} = a_1^cE_2. \tag{31}$$

Equations (24), (27) and (28), (31) may be solved for a_1^c. This constant has the value

$$a_1^c = \frac{\mu(H_2C_2 - G_2D_2)(A_1F_1 - B_1E_1)}{(\mu A_2H_2 - E_2D_2)(C_1F_1 - \mu B_1G_1) + (\mu A_2G_2 - E_2C_2)(\mu B_1H_1 - D_1F_1)} \tag{32}$$

where

$$\mu = \frac{\mu_2}{\mu_1}. \tag{33}$$

Accordingly, $\mu = 1$ if the shell is nonmagnetic and is immersed in free space. Similarly, (25), (26) and (29), (30) may be solved for b_1^c. This constant has the value

$$b_1^c = \frac{\mu k^2(C_2H_2 - D_2G_2)(A_1F_1 - B_1E_1)}{(k^2A_2G_2 - \mu C_2E_2)(k^2B_1H_1 - \mu F_1D_1) + (k^2A_2H_2 - \mu D_2E_2)(\mu F_1C_1 - k^2B_1G_1)} \tag{34}$$

where

$$k = \frac{k_2}{k_1}. \tag{35}$$

Equations (32) and (34) may now be substituted into (19) and (18), respectively, to obtain the cavity fields. It is important to note that no restrictions on the cavity size have been introduced into the theory.

In the numerical work it is of convenience to have available exact expressions for the functions appearing in (23). These are[3]

$$\left\{\begin{array}{ll} j_1(z) = \dfrac{\sin z}{z^2} - \dfrac{\cos z}{z} & [zj_1(z)]' = \left(1 - \dfrac{1}{z^2}\right)\sin z + \dfrac{1}{z}\cos z \\ h_1^{(2)}(z) = -\dfrac{1}{z}\left(1 - \dfrac{j}{z}\right)e^{-jz} & [zh_1^{(2)}(z)]' = \dfrac{1}{z}\left(1 - \dfrac{j}{z}\right)e^{-jz} + je^{-jz} \\ h_1^{(1)}(z) = -\dfrac{1}{z}\left(1 + \dfrac{j}{z}\right)e^{jz} & [zh_1^{(1)}(z)]' = \dfrac{1}{z}\left(1 + \dfrac{j}{z}\right)e^{jz} - je^{jz} \end{array}\right\}. \tag{36}$$

The Form of the Integrals To Be Evaluated by a Computer

The description of the incident electric field pulse assumed in this paper is

$$e_0(t) = A\epsilon^{-t^2/2t_1^2} \tag{37}$$

where A is the value of $e_0(0)$ in volts/m, t is the time, and t_1 is a measure of the pulse width. A is taken to be one volt/m. The spectrum of the pulse described by (37) is

$$E_0(f) = At_1\sqrt{2\pi}\,\epsilon^{-f^2/2f_1^2}. \tag{38}$$

Here f is the frequency in c/s, and $f_1 = 1/2\pi t_1$.

Let $G(f) = G_R(f) + jG_I(f)$ represent one of the desired steady-state shielding ratios, such as $[E_c(f)]_{R=0}/E_0(f)$. The time history of the electric field at the center of the cavity is then

$$e_c(t) = \int_{-\infty}^{\infty} G(f)E_0(f)\epsilon^{j2\pi ft}df$$
$$\sim 2At_1\sqrt{2\pi}\int_0^{f_c} \cdot [G_R(f)\cos 2\pi ft - G_I(f)\sin 2\pi ft]\epsilon^{-f^2/2f_1^2}df \tag{39}$$

where in obtaining the second part of (39) use has been made of the relation $G^*(f) = G(-f)$. A similar integral holds for the case of the magnetic field. In this paper the highest significant frequency contained in a Gaussian pulse is taken to be $f_c = 2.6f_1$. The "significant" base width of the time function is $2 \times 2.6t_1 = 5.2t_1$. The half-amplitude width of a Gaussian pulse is $2.355t_1$. Observe that the time history of the incident magnetic field $h_0(t)$ is given by (37) with A replaced by A/ζ ($\zeta = \omega\mu_2/k_2$).

Numerical Results

Figure 2 presents the steady-state transfer characteristic relating $E_c(f)$ to $E_0(f)$ for two aluminum spheres ($\sigma = 3.54 \times 10^7$ mhos/m) of designated wall thickness $d = a - b$. In this figure, and in subsequent figures, the solid-line curves apply to spheres of 36-inch radius, and the dashed curves apply to spheres 18 inches in radius.

Figures 3, 4, 5, and 6 present the time history of the electric field at the center of the aluminum shells for $t_1 = 6$ μs, 12 μs, 24 μs, and 48 μs, respectively.

Figure 7 shows the time history of the electric field at the center of a shielded room having the shape of a sphere. The volume of the room is 1000 cubic feet, so that its radius is 6.204 feet. The shield is copper sheet ($\sigma = 5.8 \times 10^7$ mho/m) 0.06408-inch thick (the d ameter of AWG No. 14 wire).

Figure 8 presents the steady-state transfer characteristic relating $H_c(f)$ to $E_0(f)$. If the reference had been $H_0(f)$ instead of $E_0(f)$, these curves would have been shifted upward by 51.53 dB, (20 $\log_{10} 120\pi$).

Figures 9, 10, 11, and 12 present the time history of the magnetic field at the center of the aluminum shells for $t_1 = 6$ μs, 12 μs, 24 μs, and 48 μs, respectively.

Figure 13 corresponds to Fig. 7, except that the time history of the magnetic field is presented.

[3] P. M. Morse, *Vibration and Sound*. 2nd ed. New York: McGraw-Hill, 1948, pp. 316–317.

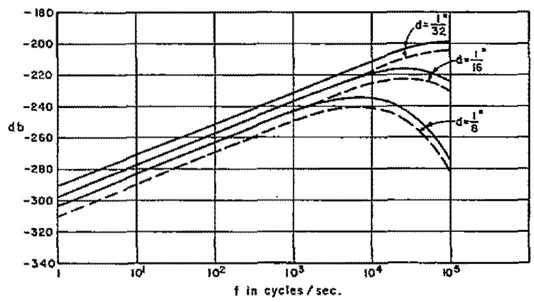

Fig. 2. Aluminum spherical shell. Steady-state transfer characteristic relating $E_c(f)$ to $E_0(f)$.

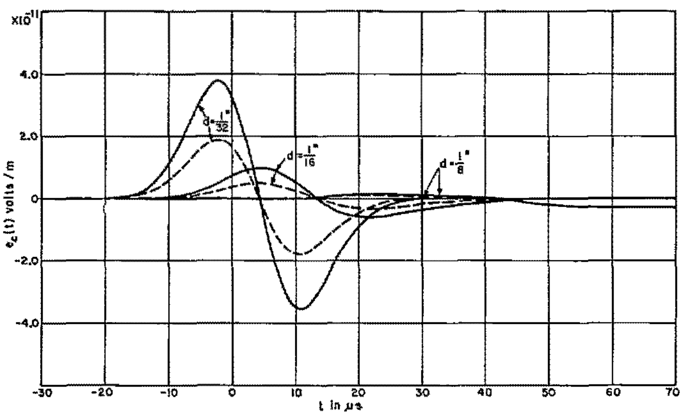

Fig. 3. Aluminum spherical shell. $t_1 = 6$ μs.

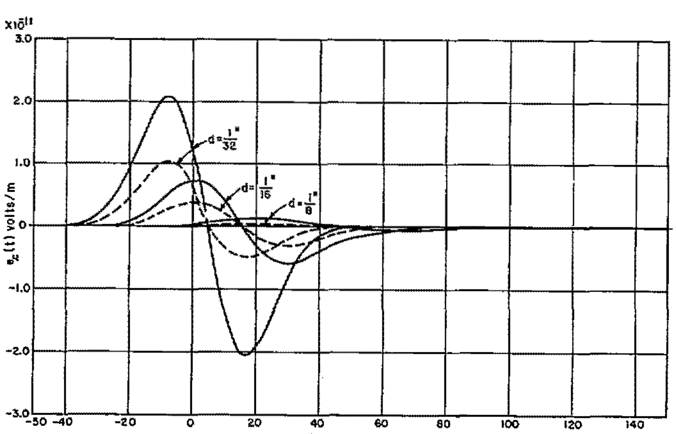

Fig. 4. Aluminum spherical shell. $t_1 = 12$ μs.

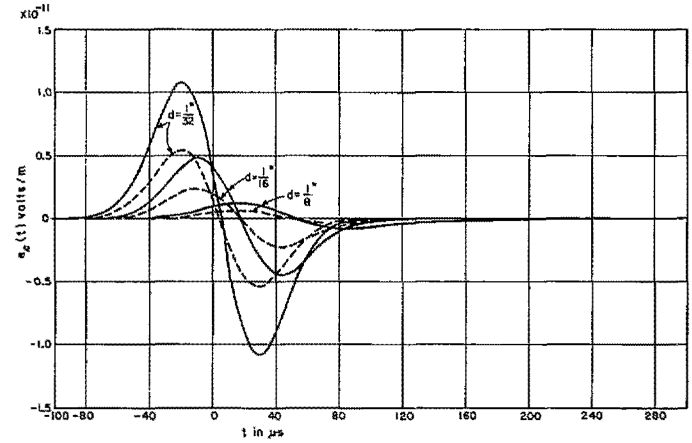

Fig. 5. Aluminum spherical shell. $t_1 = 24$ μs.

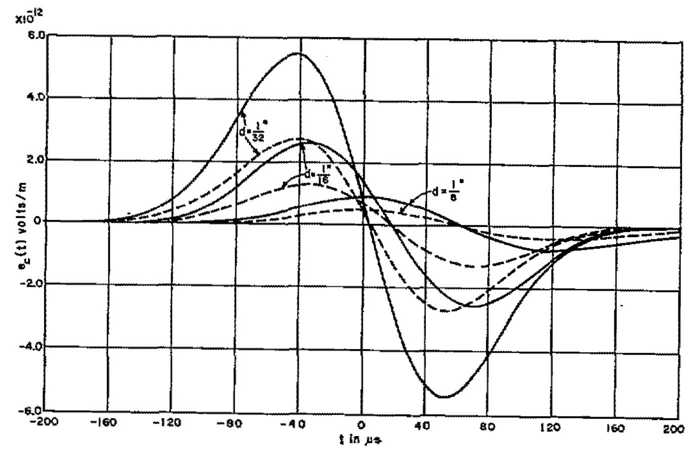

Fig. 6. Aluminum spherical shell. $t_1 = 48$ μs.

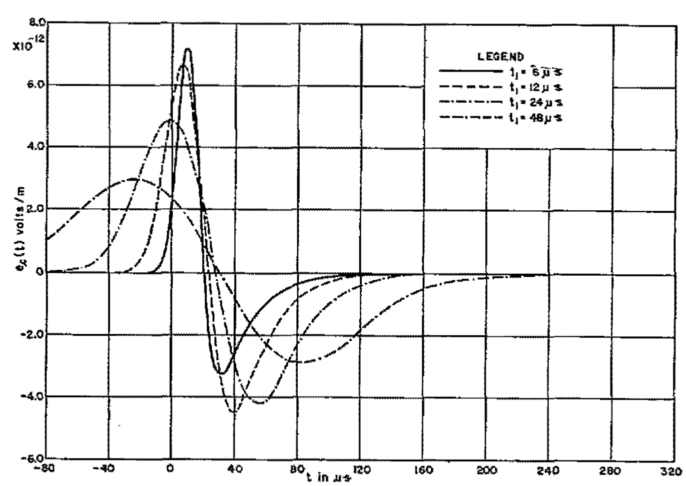

Fig. 7. Copper spherical shell (shielded room).

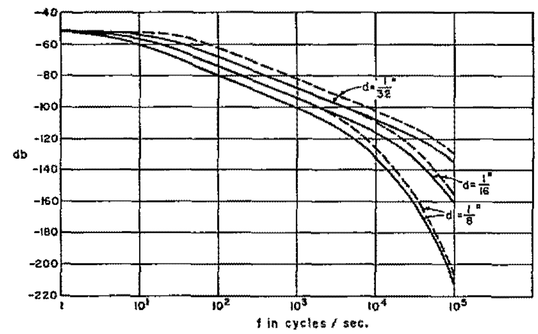

Fig. 8. Aluminum spherical shell. Steady-state transfer characteristic relating $H_c(f)$ to $E_0(f)$.

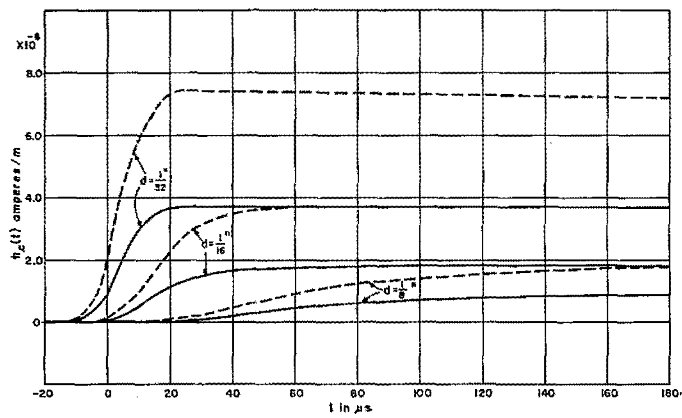

Fig. 9. Aluminum spherical shell. $t_1=6$ μs.

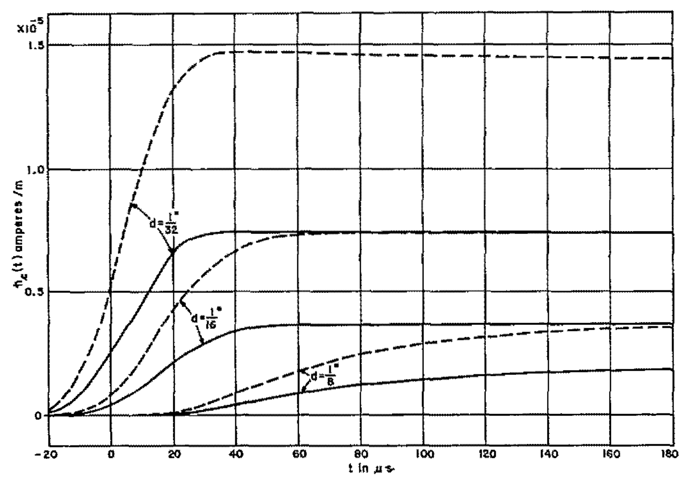

Fig. 10. Aluminum spherical shell. $t_1=12$ μs.

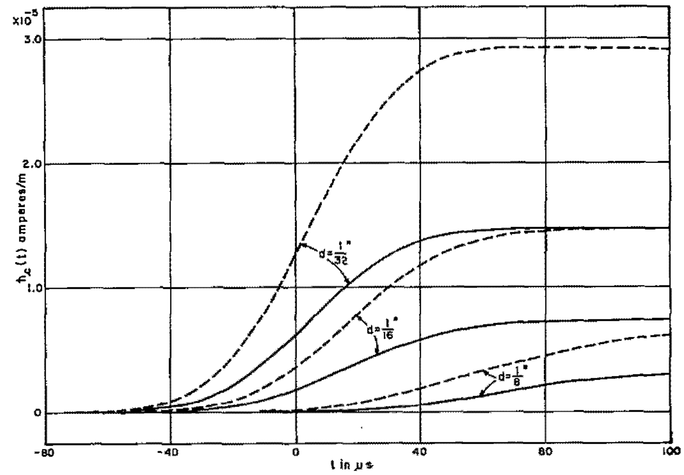

Fig. 11. Aluminum spherical shell. $t_1=24$ μs.

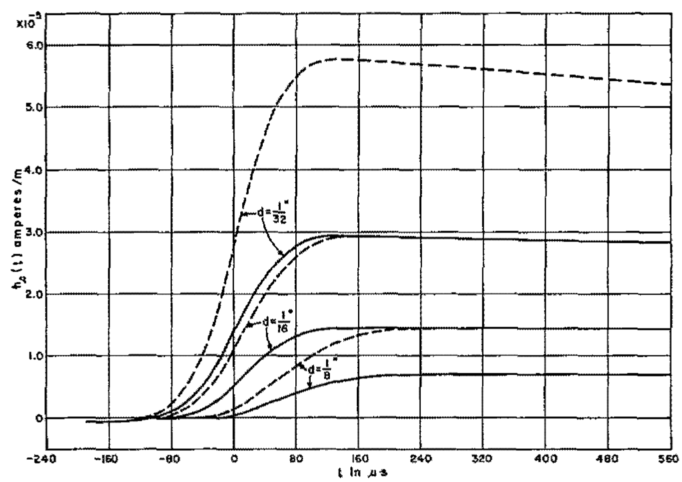

Fig. 12. Aluminum spherical shell. $t_1=48$ μs.

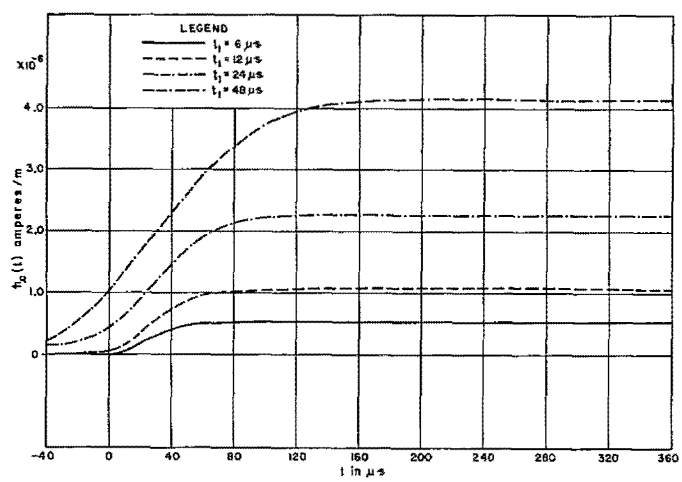

Fig. 13. Cooper spherical shell (shielded room).

Discussion

The numerical results obtained from this exact theory of spherical shields are compatible with the results reported elsewhere based on an approximate analysis of magnetic-field shielding by a spherical shell and electric-field shielding by a cylindrical shell of finite length.[4]

The following figures and graphs may be compared:

Figure 2 (present paper)	Graph 26 (footnote 4)
Figures 3, 4, 5	Graphs 27, 28, 29
Figure 8	Graph 12
Figure 11	Graph 13
Figure 12	Graph 14

Note that the results obtained in the two papers for the time histories of the magnetic field within spherical shells differ somewhat. This discrepancy is attributed to the fact that the prior work[4] is based on an approximate transfer function.[5] No previous study was made of the shielding of an electric field by a spherical shell. Intuitively one feels that the time histories of the electric field in shells of arbitrary shape made of the same material and having the same wall thickness should be qualitatively the same, provided their dimensions are small in terms of the wavelength of the incident radiation. A comparison of the curves and graphs in the two reports, as suggested above, reveals that this statement is true.

Conclusion

Insofar as the authors are aware, this is the first presentation of an exact theory relating the electric and magnetic fields at any point within an imperfectly conducting spherical shell to the incident plane-wave electric field. Using Fourier transform techniques, the time histories of the electric and magnetic fields were computed at the center of several shields for incident electric fields having the shape of a Gaussian pulse. It should be noted that the center of a shell is a point of symmetry, and, accordingly, the amplitudes and time sequences of fields at other points within the shell could be considerably different, even though the shield is electrically small at the highest significant frequency contained in the incident pulse. Suffice to say an "analytical probing" of the field at several points within and near the shield (as was recently carried out for two parallel infinite plates[6]) would be a prodigious undertaking, even considering the availability of the most modern high-speed digital computer.

Acknowledgment

The mathematical development was verified by R. S. Satterwhite. Margaret Houston programmed this problem for the Sandia Laboratory CDC-3600 Computer. The expert typing was done by Barbra A. Ford.

[4] C. W. Harrison, Jr., "Transient electromagnetic field propagation through infinite sheets, into spherical shells, and into hollow cylinders," *IEEE Trans. on Antennas and Propagation*, vol. AP-12, pp. 319–334, May 1964.

[5] L. V. King, "Electromagnetic shielding at radio frequencies," *Phil. Mag.*, vol. 15, no. 97, pp. 201–223, February 1933. Observe that the author does not state whether he computes the field at the center of the sphere or elsewhere.

[6] C. W. Harrison, Jr., M. L. Houston, R. W. P. King, and T. T. Wu, "The propagation of transient electromagnetic fields into a cavity formed by two imperfectly conducting sheets," *IEEE Trans. on Antennas and Propagation*, vol. AP-13, pp. 149–158, January 1965.